# Near-threshold $K^+$ production in Heavy-ion Collisions

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# Abstract

Within a hadronic transport model we study in detail contributions to kaon yields and momentum spectra from various baryon (resonance)-baryon (resonance) and  $\pi N$  interactions in heavy-ion collisions at beam energies near the free-space kaon production threshold. It is found that the finite lifetime of baryon resonances affects significantly the shape of kaon spectra, and the high energy parts of the kaon spectra are dominated by kaons from  $\pi N \to \Lambda K^+$  processes.  $N^*(1440)$  resonances are found to contribute about 10% to the kaon yield. Effects of boosting the Fermi momentum distributions of the two colliding nuclei into their center of mass frame, centrality of the reaction as well as the nuclear equation of state on kaon yields and spectra are also discussed. Model calculations on  $K^+$ ,  $\pi^+$  and  $\pi^-$  spectra for the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV are compared with the experimental data from the KaoS collaboration.

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#### I. MOTIVATION

Kaons have long been proposed as one of the most promising messengers of the primary violent stage of relativistic heavy-ion collisions [1]. Based on nuclear transport model calculations, it was further demonstrated that kaon production in heavy-ion collisions at subthreshold energies may be very useful in pinning down the nuclear equation of state [2]. Moreover, it has been shown recently that kaons are also very useful in studying in-medium properties of hadrons in the hot and dense zone formed in the reaction [3]. However, due to the complexity of the reaction dynamics and uncertainties in elementary kaon production processes, the physics extracted from kaon data is still very limited despite extensive efforts in the past years. For a recent review of the experimental and theoretical study on kaons, we refer the reader to refs. [4,5].

The most recent data on kaon production from the reaction of Au+Au at a beam energy of 1.0 GeV/nucleon taken by the KaoS collaboration [6] at SIS/GSI has further stimulated much interest and theoretical work on the mechanism of kaon production, the extraction of the nuclear equation of state and in-medium properties of kaons. Most of these studies are based on transport models [7,8] or quantum molecular dynamics [9,10]. However, due to large discrepancies among the results of different model calculations, the interpretation of the experimental data has been difficult. To better understand the discrepancies among previous model calculations and most importantly to help interpretating the experimental data more accuruately, we perform a study on several aspects of kaon production using another independent transport model for relativistic heavy-ion collisions detailed in refs. [11,12]. We claim, by no means, that the model used here is superior to any model used previously in studying kaon production. But the aspects discussed in the following are useful for improving some of the previous calculations.

Firstly, in all of the previous calculations, one either uses the frozen-resonance approximation (assume resonances have infinite lifetime) or let the resonances decay but assume that kaons from  $\pi N$  collisions can be neglected on the ground that  $\pi N$  collisions only con-

tributate about 25% to the total kaon yields [13–15]. It was then concluded that the kaon yield in heavy-ion collisions at beam energies around 1.0 GeV/nucleon is dominated by  $N\Delta$ ,  $\Delta\Delta$  and NN collisions sequentially. However, it has been shown by several authors that the production dynamics, multiplicity and spectra of pions cannot be properly described by using the frozen-resonance approximation [11,16–18]. Since kaons calculated perturbatively in all of the models are almost completely determined by the  $\pi$ , N,  $\Delta$ ,  $N^*$  dynamics, it is therefore worth studying how the conclusions reached earlier may be affected by the finite lifetime of baryon resonances. Indeed, we found that although  $\pi N$  collisions only contributate about 25% to the total kaon yield, it is comparable to contributions of most individual baryon-baryon collisions and should not be negelected. The contribution from  $\pi N$  collisions actually dominates the high energy part of kaon spectra. The shapes of kaon spectra calculated using infinite or finite lifetimes for baryon resonances differ significantly.

Secondly, at beam energies around 1.0 GeV/nucelon  $N^*(1440)$  resonances can also be excited. Effects of the  $N^*$  resonance on kaon yields and spectra have not been studied. We therefore also included in the present study contributions to kaons from the  $N^*$  involved collisions. It is found that the  $N^*$  involved collisions only contribute about 10% to the total kaon yield and its effect on kaon spectra is minor.

Thirdly, in view of the fact that some of the previous studies (e.g. ref. [9]) used the non-relativistic transformation to boost the Fermi momentum distributions of the two colliding nuclei into their center of mass frame, we compare kaon yields and spectra calculated using the relativistic and the non-relativistic momentum transformation. This seemingly numerical treatment has vitally important physical consequence. Not only the relative contributions from NN and other collision channels differ dramatically, the total kaon yield and spectrum change as large as that caused by changing from the soft nuclear equation of state (with an incompressibility K=200 MeV) to the stiff one (K=380 MeV).

Finally, since the dynamics of kaon production is virtually determined by the  $\pi$ , N,  $\Delta$ ,  $N^*$  dynamics, especially in the perturbative method used in calculating kaons, and it is well known that pion observables provide strong constraints on the reaction dynamics, we study

also whether pions from the same reaction can be well described simultaneously as kaons by comparing  $K^+$ ,  $\pi^+$  and  $\pi^-$  spectra with the experimental data from the KaoS collaboration.

The paper is organized as follows. In the next section necessary inputs for calculating kaons are presented. Our results will be presented and discussed in section 3. A summary will be given at the end.

#### II. THE MODEL AND INPUTS

In the present study we use the hadronic transport model for relativistic heavy-ion collisions [11,12]. It is based on the numerical solution of a coupled set of transport equations for the phase space distribution functions of nucleons, baryon resonances ( $\Delta(1232)$ ) and  $N^*(1440)$ ) and pions [19]. The model has been rather successful in studying several aspects of relativistic heavy-ion collisions.

As for the production of kaons, we use the perturbative method first used by Randrup and Ko in ref. [1] and nowadays widely used by others. We include kaon production channels due to both baryon-baryon collisions (i.e.  $B_1B_2 \to BYK^+$ , here B represents a baryon and Y represents a hyperon.) as well as pion-nucleon collisions (i.e.  $\pi N \to \Lambda K^+$ ). More specifically, we include kaons from the interaction of  $NN, N\Delta, \Delta\Delta, NN^*, \Delta N^*, N^*N^*$  and  $\pi N$ .

For kaons from baryon-baryon collisions, we use the standard Randrup-Ko parameterizations for both the total and the differential kaon production cross section [1]. For completeness we copy these parameterizations in the following. The kaon production cross section from a nucleon-nucleon interaction was parameterized as

$$\sigma_{NN \to BYK^+}(\sqrt{s}) = 36 \frac{p_{\text{max}}}{m_K} \mu \text{b},$$
 (1)

where the maximum kaon momentum  $p_{\text{max}}$  is related to the nucleon-nucleon center-of-mass energy  $\sqrt{s}$  by

$$p_{\text{max}} = \frac{1}{2} \sqrt{[s - (m_B + m_Y + m_K)^2][s - (m_B + m_Y - m_K)^2]/s}.$$
 (2)

Kaon production cross sections from a nucleon-delta and a delta-delta interaction were parameterized respectively as

$$\sigma_{N\Delta \to BYK^+}(\sqrt{s}) \approx \frac{3}{4} \sigma_{NN \to BYK^+}(\sqrt{s}),$$
 (3)

$$\sigma_{\Delta\Delta\to BYK^+}(\sqrt{s}) \approx \frac{1}{2}\sigma_{NN\to BYK^+}(\sqrt{s}).$$
 (4)

For the  $N^*$  involved collisions we assume that the kaon production cross sections are the same as that in the  $\Delta$  involved collisions.

In addition to the total kaon production cross section, the kaon momentum distribution from the baryon-baryon interaction was also parameterized by Randrup and Ko according to the phase space argument,

$$\frac{E}{p^2} \frac{d^3 \sigma(\sqrt{s})}{dp d\Omega} = \sigma_{K^+}(\sqrt{s}) \frac{E}{4\pi p^2} \frac{12}{p_{\text{max}}} \left(1 - \frac{p}{p_{\text{max}}}\right) \left(\frac{p}{p_{\text{max}}}\right)^2, \tag{5}$$

where  $\sigma_{K^+}$  is the total kaon production cross section and  $p_{\text{max}}$  is the maximum kaon momentum.

For kaons from pion-nucleon collisions, we adopt the parameterization by Cugnon [13]. The total kaon production cross section was parameterized by averaging over the processes  $\pi^0 p \to K^+ \Lambda$  and  $\pi^+ n \to K^+ \Lambda$  and can reproduce quite accurately the available data up to  $\sqrt{s} = 3.0$  GeV. The cross section reads

$$\sigma_{\pi N \to K^+ \Lambda}(\sqrt{s}) = 2.47 \cdot (\sqrt{s} - \sqrt{s_0}) \ mb \tag{6}$$

for  $\sqrt{s_0} = m_\Lambda + m_K < \sqrt{s} < 1.7 \text{ GeV},$ 

$$\sigma_{\pi N \to K^+ \Lambda}(\sqrt{s}) = \frac{0.0225}{\sqrt{s} - 1.6} \ mb$$
 (7)

for  $\sqrt{s} > 1.7$  GeV. The angular distribution of kaons in the  $\pi N$  center of mass system is taken as isotropic. Consequently, the Lorentz invariant double differential kaon production cross section can be written as

$$\frac{E}{p^2} \frac{d^3 \sigma(\sqrt{s})}{dp d\Omega} = \sigma_{K^+}(\sqrt{s}) \frac{E}{4\pi p^2} \delta(p - q), \tag{8}$$

where q is the kaon momentum in the  $\pi N$  center of mass system which is uniquely determined by the energy-momentum conservation.

With the above elementary kaon production cross sections, both the total kaon production probability and the kaon Lorentz-invariant double differential cross section in heavy-ion collisions can be calculated in the standard way. In particular, the contribution to the kaon spectrum in the laboratory frame from each elementary collision is calculated analytically according to Eqs. (5) and (8) by performing successively two Lorentz transformations.

#### III. RESULTS AND DISCUSSION

In the following we present and discuss results of our calculations on kaon production probabilities and spectra. We concentrate on the four aspects discussed in the introduction. It should be mentioned that the momentum-independent, Skyrme-type mean field is used for all kinds of baryons. Because kaon rescatterings have been found to affect mainly the spectrum at large angles [20,8], and since we have in mind the kaon spectrum measured at  $\theta_{lab} = 44^{\circ}$  in the reaction of Au+Au at  $E_{beam}/A=1.0$  GeV, kaon rescatterings are negelected in the present study.

## A. Sources of kaons in heavy-ion collisions

An important question concerning kaon production in heavy-ion collisions is where the kaons come from. For reactions at beam energies around 1.0 GeV/nucleon, using the frozen-resonance approximation it was shown previously that both the kaon yields and spectra are dominated by  $N\Delta$ ,  $\Delta\Delta$  and NN collisions sequentially. To see how this conclusion may be affected by the finite lifetime of baryon resonances we compare in parts (A) and (B) of Fig. 1 kaon production probabilities from various collision channels calculated using the experimental, energy dependent resonance widths (A) with those calculated using the frozen-resonance approximation (B). The calculation was done for the reaction of Au+Au at a beam energy of 1.0 GeV/nucleon and an impact parameter of 1.0 fm. The soft nuclear

equation of state corresponding to an incompressibility K = 200 MeV is used. In both cases the Lorentz transformation is used to boost the initial momentum distributions of the two colliding nuclei into their center of mass frame.

There are several interesting points to be noticed. First, we notice that the total kaon production probabilities in the two calculations are comparable ( $P_A = 0.0653$  and  $P_B = 0.0686$ ). The main difference lies in the relative contributions to the kaon yield and the shapes of the kaon spectrum. The kaon production probabilities from both the  $N\Delta$  and the  $\Delta\Delta$  collision channel are reduced by about 50% when finite lifetimes are used for the resonances. The  $\pi N$  contribution is about 25% of the total kaon yield. This is in agreement with previous studies [13–15]. However, it is seen that the  $\pi N$  contribution is comparable to contributions of most baryon-baryon collision channels and needs to be included. As we will discuss later, the  $\pi N$  contribution to the kaon spectrum is significantly different from that of baryon-baryon collisions due to the rather different energy dependence of the elementary kaon production cross sections and the reaction kinematics.

Second, the kaon production probabilities from the  $N^*$  involved collisions is increased when finite lifetimes are used for baryon resonances. This is because the  $N^*$  resonance is mainly excited through  $\pi N$  collisions, while the excitation through NN collisions is negligibly small at beam energies around 1.0 GeV/nucleon.

Third, we notice that the contribution from  $\Delta\Delta$  collisions is small compared to that from NN collisions even in the case of using the frozen-resonance approximation. This is in contrast to some of the previous calculations where the contribution from  $\Delta\Delta$  dominates over that from NN collisions [9,10]. The origin of this difference remains to be explored. One of the possible reasons is the use of the non-relativistic momentum transformation in some of the previous studies.

In view of the fact that the non-relativistic transformation was used in some of the previous calculations to boost the initial Fermi momentum distributions of the two colliding nuclei into their center of mass frame, we study now effects of the momentum transformation. The importance of the nuclear Fermi motion in subthreshold particle production is well

konwn. The free threshold beam energy for kaon production is 1.58 GeV; it is reduced to about 0.8 GeV after taking into account the Fermi motion [2]. In transport models or quantum molecular dynamics models, nuclear reactions are simulated in the nucleus-nucleus center of mass frame. The boosting of the two Fermi spheres has to be treated with care. At relativistic energies, the boost must be done with the Lorentz transformation. For example, for the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV, the Lorentz  $\gamma$  factor is 2.1, the non-relativistic transformation supresses the kinetic energy of each nucleon in the nucleus-nucleus center of mass frame by about 35 MeV on the average.

To be more quantitative, we presnt in parts (C) and (D) of Fig. 1 the kaon production probabilities calculated using the non-relativistic momentum transformation. Results in (C) are calculated with finite lifetimes of baryon resonances and that in (D) are calculated with the infinite lifetime. The total kaon production probability in case C is  $P_C = 0.0453$  and that in case D is  $P_D = 0.0406$ . Comparing them with results shown in (A) and (B), it is seen that the total kaon yields decrease by about 40% when the non-relativistic momentum transformation is used. In particular, the kaon yield from NN collisions calculated with the relativistic transformation is about twice that calculated using the non-relativistic transformation. We notice that the change of the kaon yield due to the momentum transformation is comparable to that caused by changing the nuclear equation of state. This can be seen from Fig. 2 where the kaon production probability calculated using an incompressibility K=380 MeV and the relativistic momentum transformation is shown. In this case the total kaon production probability is P = 0.0360. It is seen that the total kaon yield is reduced by about 45% by changing the incompressibility from K=200MeV to 380 MeV. It is therefore clear that the seemingly numerical treatment of the initial momentum transformation has an important physical consequence. Kaon yields calculated previously using the non-relativistic transformation for the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV should be increased correspondingly.

Comparing results calculated using the finite and infinite lifetimes for baryon resonances, it is seen that the total kaon yield changes only by about 5% to 12% although the relative

contributions from the various collision channels change significantly. The shape of the kaon spectrum, however, varies significantly. The kaon spectra calculated in the laboratory frame are shown in Fig. 3 for the cases A, B, C and D. It is noticed that the frozen-resonance approximation underestimates the high momentum part of the kaon spectra. This is easily understandable. On the average the center of mass of a  $\pi N$  pair moves faster than that of a baryon-baryon pair due to the lighter mass. The kaon from the low threshold ( $\sqrt{s} = 1.61$  GeV) but two-body final state  $\pi N \to \Lambda K$  process has the same average energy as the kaon from the high threshold ( $\sqrt{s} = 2.55$  GeV) but three-body final state  $BB \to BYK$  process. Therefore, the kaon from  $\pi N$  collisions can have higher laboratory momenta than the kaon from baryon-baryon collisions.

# B. Centrality dependence of kaon production

In this section we concentrate on the centrality dependence of the total kaon production probabilities. In Fig. 4 we present kaon production probabilities from various channels as a function of impact parameter for both the soft (left) and the stiff (right) equations of state. The rapid decrease of the kaon production probability with the increasing impact parameter is obvious. By changing from the soft nuclear equation of state to the stiff one, the total kaon production probability changes by about 40% to 50% in the whole impact parameter range. The most significant change happens to the relative contributions from the  $N\Delta$  and NN collisions. With the soft equation of state, the contribution from  $N\Delta$  collisions is slightly higher than that from NN collisions in central collisions. The two contributions are comparable in peripheral collisions. While with the stiff equation of state, the contribution from NN collisions dominates over the contribution from  $N\Delta$  collisions. The contribution to kaons from the NN collisions is reduced by about 27%, while that from the  $N\Delta$  collisions is reduced by about 44% comparing to the calculation with the soft equation of state. The overall reduction of the kaon yields is due to the lower compression reached in the reaction and the smaller energy available for particle production or excitation

of nucleons with the stiff equation of state. Furthermore, since the production of kaons from  $N\Delta$  collisions requires two successive interactions  $(NN \to N\Delta \text{ and } N\Delta \to NYK)$ , kaons from the  $N\Delta$  collisions are more sensitive to the change of the nuclear equation of state.

## C. Effects of $N^*(1440)$ resonances on kaon production

Experimental data on particle production accumulated at several laboratories during the last decade indicate that a gradual transition to resonance matter occurs in the participant region of heavy-ion collisions at beam energies of 1 to 2 GeV/nucleon [5]. To study in detail properties of this new form of matter it is important to determine its baryonic composition, i.e., the relative populations of nucleons and various baryon resonances. In the 1-2 GeV/nucleon beam energy range, the important baryonic resonances are  $\Delta(1232)$ ,  $N^*(1440)$ ,  $N^*(1520)$  and  $N^*(1535)$ . The excitation of the  $\Delta(1232)$  and  $N^*(1535)$  resonances have been studied extensively through the production of pions and etas respectively [21,22]. The study on the  $N^*(1440)$  resonance has been rather rare although it is the second most important baryon resonance to be excited in the energy range considered here. Here we discuss briefly effects of the  $N^*(1440)$  resonances on kaons, more detailed study of the  $N^*$  resonance together with its effects on subthreshold antikaon and antiproton production will be published elsewhere [23].

First, to evaluate the importance of the excitation of the  $N^*(1440)$  resonance we show in Fig. 5 the  $N^*(1440)$  multiplicity during the Au+Au collisions at beam energies of 1 to 2 GeV/nucleon and impact parameters between 1 and 9 fm. It is seen that the population of the  $N^*(1440)$  resonance strongly depends on the impact parameter and the beam energy. In central collisions the maximum number of  $N^*(1440)$  resonances increases from 7 to 17 as the beam energy increases from 1 to 2 GeV/nucleon. For comparison, it should be mentioned that the maximum number of the  $\Delta(1232)$  resonance increases from 52 to 89 in the same collisions and therefore the ratio of the populations of the  $N^*$  and the  $\Delta$  increases from 14% to 19% as the beam energy increases from 1 to 2 GeV/nucleon.

In Fig. 6, we show the total and the  $N^*$  induced kaon production probabilities in Au+Au collisions at beam energies between 1 and 2 GeV/nucleon and impact parameters between 1 and 9 fm. The total kaon production probabilities are shown with the solid lines while the contributions from the  $N^*$  involved collisions are shown with the dashed lines. It is seen that, the  $N^*$  involved collisions contribute only about 11% in the whole energy and impact parameter ranges. This result is not surprising since baryon-baryon collisions at center of mass energies around the kaon production threshold of  $\sqrt{s} = 2.55$  GeV are dominated by the production and absorption of  $\Delta(1232)$  resonances. Effects of the  $N^*$  resonance on the kaon spectrum will be discussed later.

# D. Pion spectra

From what we have discussed above, it is clear that the production of kaons is virtually determined by the  $\pi$ , N,  $\Delta$ ,  $N^*$  dynamics of the reaction. Since kaons are calculated perturbatively during the reaction, to extract reliably the interesting physics from kaon observables it is a prerequisite that the reaction dynamics is correctly described. This can be done by studying pion observables, such as pion spectra. For this purpose we discuss in this section the pion spectra. Fortunately, the kaoS collaboration have measured both the kaon and pion spectra in the same experiments [4,6].

It is well known that pion spectra are rather insensitive to the nuclear equation of state since the total pion multiplicity calculated with different equations of state differs only by about 15%. Here we show pion spectra calculated with the soft equation of state. In Fig. 7 and Fig. 8 we compare the calculated  $\pi^+$  and  $\pi^-$  spectrum with the experimental data measured at  $\theta_{lab} = 44^{\circ}$  in the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV. The calculations are shown by open squares with error bars. The available experimental data from the KaoS collaboration are given by the solid circles [4,6,24]. It is seen that the data can be very well reproduced. For completeness and future comparison we present the model prediction of  $\pi^0$  spectrum in Fig. 9. The comparison between the model calculation and the experimental

data on pion spectra indicates that the complicated reaction dynamics is well described in the model.

#### E. Kaon spectra

We now turn to our study on kaon spectra by comparing the kaon spectra from different collision channels and performing a comparison between the model calculation and the experimental data.

Kaon spectra from the seven collision channels  $(N\Delta, NN, \Delta\Delta, NN^*, N^*N^*, \Delta N^*$  and  $\pi N)$  calculated at  $\theta_{lab} = 44^0$  in the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV are displayed in Fig. 10. The right window shows results calculated using the soft nuclear equation of state while the left window shows that calculated using the stiff equation of state. It is interesting to note that in the low momentum part  $(P_{lab} \leq 0.5 \text{ GeV/c}) N\Delta$  and NN collisions dominate, while in the high momentum part  $\pi N$  and  $NN^*$  collisions dominate. The fact that  $\pi N$  collisions are the most important source for high momentum kaons is mainly due to the reaction kinematics as we have explained early. It should also be mentioned that the kaon spectra from  $\pi N$  collisions have larger statistical fluctuations due to the resonance like  $\pi N \to \Lambda K$  cross section. It is also seen that the overall effect of the  $N^*$  resonace on kaon spectra is small. However, it is interesting to mention that the contribution from  $NN^*$  collisions is much larger than that from  $\Delta \Delta$  collisions, especially in the high energy part of the spectra. This is mainly due to the large  $\rho_N/\rho_\Delta$  ratio and the higher mass of the  $N^*$  resonances.

We now perform a comparison between the model calculation and the experimental data in Fig. 11. The experimental kaon spectrum measured at  $\theta_{lab} = 44^{\circ}$  in the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV is plotted with solid circles. The open circles and squares are calculated with the soft and stiff equations of state respectively. It is seen that the experimental data can be well reproduced with the soft equation of state. This finding is in agreement with that of ref. [10] although the sources of kaons are quite different. The fact

that both the pion and the kaon spectrum can be well reproduced simultaneously indicates that the production of kaons is mainly determined by the  $\pi, N, \Delta, N^*$  dynamics of the reaction and it has been well accounted for in the model, in addition the elementary cross sections used for kaon production are reasonable.

It should, however, be stressed that the agreement between the calculation and the data cannot be used to extract the nuclear equation of state since the rescattering of kaons although its effect is samll at this angle, and moreover, neither the momentum dependent part of the mean field nor the in-medium effects on kaons and nucleons are taken into account in the present model calculations. It has been shown that the momentum dependent interaction reduces the kaon yield by a factor of 2 to 3 [10]. On the other hand, more consistant RBUU calculations includeing both the momentum-dependent mean field and the medium effects have shown that the medium effects on kaons results in an enhancement of kaon yields by about a factor of 2 to 3 [8]. It would therefore be interesting to study in the future whether the model used here can still reproduce the data with both the momentum dependent mean field and the in-medium effects. However, due to the still rather model dependent prescription of the momentum dependent mean field and the in-medium effects it has been difficult to compare results from different groups and to extract reliably the interesting physics from the kaon data. It seems therefore plausible to first compare calculations with the least and widely accepted model approximations. The present model calculations and associated discussions on several basic aspects of kaon production are useful in this respect.

#### IV. SUMMARY

Using a hadronic transport model we have studied several aspects of near-threshold kaon production in relativistic heavy-ion collisions. We showed that the finite lifetime of baryon resonances affects significantly the shape of the kaon spectra.  $\pi N \to \Lambda K$  channel is found to be the most important source for high energy kaons.  $N^*(1440)$  resonances contribute about

10% to the total kaon yield and have a minor effect on the kaon spectra. The initial Fermi momentum transformation has an effect on kaon yields and spectra as large as that of the nuclear equation of state. With the present model, both the pion and the kaon spectra from the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV can be well reproduced simultaneously with the soft nuclear equation of state.

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#### FIGURE CAPTIONS

- Fig. 1 Time evolution of kaon production probabilities in the reaction of Au+Au at  $E_{beam}/A = 1.0 \text{ GeV}$  and b=1.0 fm using the soft equation of state. (A) Using finite lifetime for resonances and the relativistic momentum transformation (B) Using infinite lifetime for resonances and the relativistic momentum transformation
- Fig. 1 continued (C) Using finite lifetime for resonances and the non-relativistic momentum transformation (D) Using infinite lifetime for resonances and the non-relativistic momentum transformation
- Fig. 2 Time evolution of kaon production probabilities for the Au+Au reaction using the stiff equation of state.
- Fig. 3 Kaon momentum spectra corresponding to the cases A, B, C and D in Fig. 1.
- Fig. 4 Impact parameter dependence of kaon production probabilities for the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV for both the soft and stiff equation of states.
- Fig. 5 Time evolution of the  $N^*(1440)$  populations in the Au+Au collisions. (left) Impact parameter dependence. (right) Beam energy dependence.
- **Fig. 6** The total (solid) and  $N^*(1440)$  induced (dashed) kaon production probabilities in the Au+Au collisions. (left) Impact parameter dependence. (right) Beam energy dependence.
- Fig. 7  $\pi^+$  spectra for the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV.
- **Fig. 8**  $\pi^-$  spectra for the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV.
- **Fig. 9**  $\pi^0$  spectra for the reaction of Au+Au at  $E_{beam}/A = 1.0$  GeV.
- Fig. 10 Kaon spectra from 7 different collision channels in the reaction of Au+Au at  $E_{beam}/A = 1.0 \text{ GeV}.$

